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TO: Dr. D. Mills
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DISCHARGE ALLOCATION OF A HYDRO-ELECTRIC GENERATING STATION - A PRELIMINARY STUDY

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This preliminary study concerns the division of discharge among individual units in a hydro-electric generating station to provide optimal power output within operating constraints. Solution approaches for the problem are proposed. A mathematical programming approach is described for the case where all the generating units in a station are non-identical. For the case where they are identical, a heuristic approach is proposed. Suggestions for further work are made.

1. INTRODUCTION

The amount of electric power that can be generated in a multi-unit hydraulic station is dependent on the amount of water discharging through each unit. This, in turn, depends on the height at which the water falls (head) and the losses during the fall/2,4,5/. The efficiency of a hydraulic generating unit, that is, the ratio of electric power to the amount of water necessary to produce it, changes with the unit's output level. This relationship is not linear. Since the efficiency characteristics may not be the same for different units, the determination of the optimal division of water flow (discharge) per second among generating units in a hydraulic station is not straightforward.

The problem of determining the division of discharge which results in maximum power output has been studied previously/1/. However, no significant results have been reported in the literature since 1969. In view of major advancements in optimization theory and computer facilities in recent years, there is a need to re-examine the situation for the purpose of obtaining improved methodology. After discussions with the Hydraulic Generation Performance Department, it was decided to carry out a preliminary investigation on the optimal discharge

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allocation for stations with identical and non-identical generating units. This report summarizes the results of the study.

2. OPTIMAL DISCHARGE ALLOCATION FOR NON-IDENTICAL UNITS

In this section, a mathematical programming model is developed to optimize the division of discharge among generating units that are non-identical.

2.1 The Model

2.1.1 Definitions

N — total number of generating units in a hydro-electric generating station

i — generating unit number, $i = 1,2,\ldots,N$

Y_i — quantity of discharge through generating unit i

M_i — maximum quantity of discharge allowed through generating unit i

E_i — quantity of discharge through generating unit i which will result in maximum efficiency

L_i — minimum quantity of discharge through generating unit i as required by operating constraints

K_i — quantity of discharge through generating unit i below which no power will be produced

$P_i(Y_i)$ — power output resulting from a discharge Y_i trough generating unit i

Q — total discharge through the station

2.1.2 Problem Formulation

The problem is to determine $(Y_1, Y_2,\ldots, Y_N)$ so that the total power output given by $\sum_{i=1}^{N} P_i(Y_i)$ is a maximum, subject to the following constraints:

$$Y_i \geq L_i \qquad i = 1,2,\ldots,N$$

$$Y_i \leq M_i \qquad i = 1,2,\ldots,N$$

$$\sum_{i=1}^{N} Y_i = Q$$

$$Y_i \geq 0 \qquad i = 1,2,\ldots,N$$

The power output of each hydraulic generating unit is actually a function of both discharge and head. However, the desired division of discharge among various generating units of the hydraulic station is always performed at a given head. Hence, the dependence of discharge on head is not shown explicitly in the formulation.

The objective function is nonlinear because the quantity of electricity generated is a nonlinear function of the water drafted through each generating unit. Figure 1 shows a typical output-discharge curve for a hydraulic generating unit. It can be shown that the relationship between power output and discharge is that of a polynomal of degree 3. At a discharge of less than K_i, no electricity will be produced. However, economic and other operating considerations normally require the unit to be operated at a discharge of at least L_i ($\geq K_i$).

2.2 Solution Approaches

A nonlinear programming software package will be needed to solve the above mathematical programming model with a nonlinear objective function. At present, no such software package is available in Ontario Hydro. They are available commercially but their suitability for this application has yet to be evaluated.

To overcome the problem of nonlinearity in the objective function, the nonlinear programming model is transformed into a linear programming (LP) model by using piecewise-linear approximation/3/, where the output-discharge curve is approximated by a set of straight lines.

Figure 1 shows that the ouptput-discharge curve is approximated by drawing straight lines through points O, A, B and C. Let $\phi_i(Y_i)$ be the resulting approximating output function based on straight lines OA, AB and BC. It can be shown/3/ that

$$\phi_i(Y_i) = P_i(E_i)X_{4i-1} + P_i(M_i)X_{4i} \qquad i = 1,2,\ldots,N$$

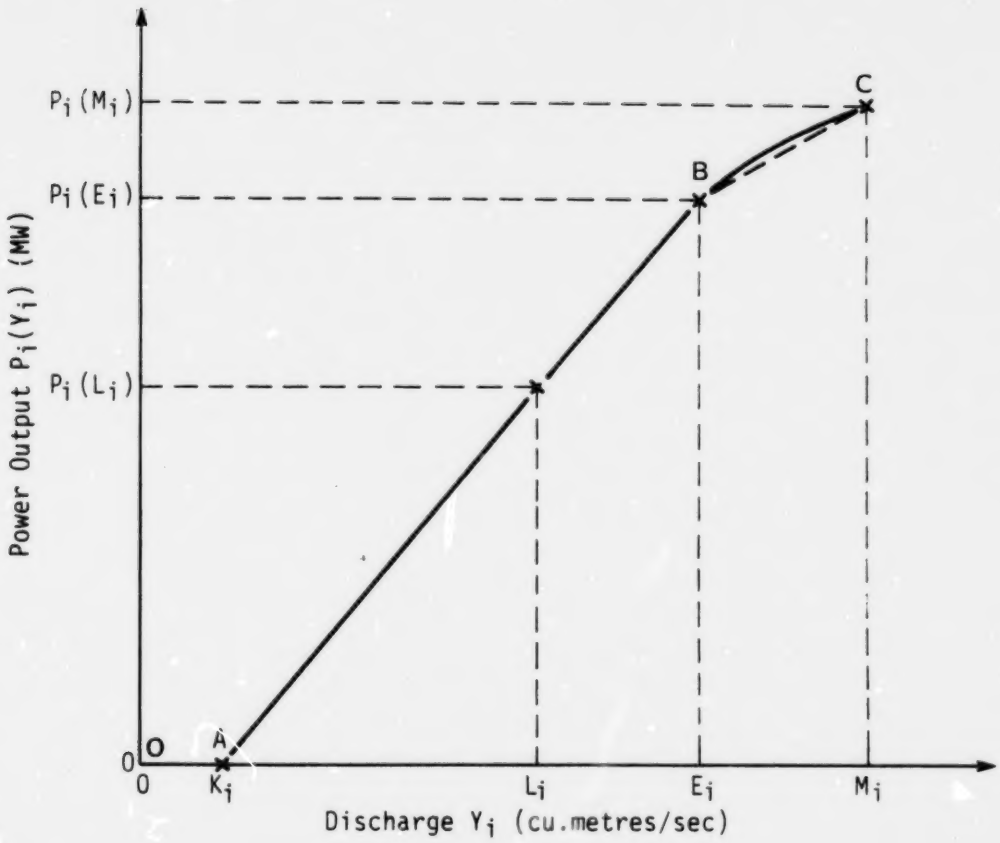

FIGURE 1

TYPICAL OUTPUT-DISCHARGE CURVE FOR A HYDRAULIC GENERATING UNIT

$$Y_i = K_i X_{4i-2} + E_i X_{4i-1} + M_i X_{4i} \qquad i = 1,2,\ldots,N$$

$$\sum_{j=0}^{3} X_{4i-j} = 1 \qquad i = 1,2,\ldots,N$$

where X_{4i-j} are auxiliary variables. Since there are four points defining the straight-line approximation in Figure 1, each discharge (Y_i) is represented by four such variables. Note that in the expression for Y_i, the term containing X_{4i-3} is missing because its coefficient is zero.

Since the $P_i(Y_i)$ function is now approximated by $\phi_i(Y_i)$, the reformulated problem is written as follows:

Determine $(X_j, j = 1,2,\ldots 4N)$ so that

$$\sum_{i=1}^{N} \{P_i(E_i)X_{4i-1} + P_i(M_i)X_{4i}\}$$

is a maximum, subject to the constraints:

$$K_i X_{4i-2} + E_i X_{4i-1} + M_i X_{4i} \geq L_i \qquad i = 1,2,\ldots,N$$

$$K_i X_{4i-2} + E_i X_{4i-1} + M_i X_{4i} \leq M_i \qquad i = 1,2,\ldots,N$$

$$\sum_{j=0}^{3} X_{4i-j} = 1 \qquad i = 1,2,\ldots,N$$

$$\sum_{i=1}^{N} (K_i X_{4i-2} + E_i X_{4i-1} + M_i X_{4i}) = Q$$

$$X_j \geq 0 \qquad j = 1,2,\ldots,4N$$

Note that $\{X_{4i-3}, X_{4i-2}, X_{4i-1}, X_{4i}\}$ constitutes a special ordered set (SOS). That is, either X_{4i}, or X_{4i-1}, or X_{4i-2}, or X_{4i-3}, or X_{4i-3} and X_{4i-2}, or X_{4i-2} and X_{4i-1}, or X_{4i-1} and X_{4i} is (are) positive.

The optimal LP solution to the reformulated problem may violate the SOS condition above. A branch-and-bound (B&B) approach is then used to determine the feasible optimal solution.

The solution of the above model will provide an approximate solution to the nonlinear programming model. The accuracy of the solution will depend on the accuracy of the piecewise-linear approximation to the output-discharge curves. As the number of generating units in the hydraulic station increases, the problem becomes increasingly difficult to solve.

2.3 Example

Consider a hydro-electric generating station consisting of 2 non-identical units, with the following characteristics obtained from their respective output-discharge curves:

M_1 = 2410 cu ft/sec, E_1 = 1420 cu ft/sec, L_1 = 110 cu ft/sec,

M_2 = 2500 cu ft/sec, E_2 = 2010 cu ft/sec, L_2 = 110 cu ft/sec,

$P_1(M_1)$ = 47.0 MW, $P_1(E_1)$ = 32.0 MW, $P_1(L_1)$ = 0.0 MW,
$P_2(M_2)$ = 55.0 MW, $P_2(E_2)$ = 46.0 MW, and $P_2(L_2)$ = 0.0 MW.

For this problem, the nonlinear programming model is written as follows:

$$\text{Maximize } P_1(Y_1) + P_2(Y_2)$$

subject to the following constraints:

$$110 \leq Y_1 \leq 2410$$

$$110 \leq Y_2 \leq 2500$$

$$Y_1 + Y_2 = 0$$

$$Y_1, Y_2 \geq 0$$

Reformulating,

$$\text{Maximize } 32X_3 + 47X_4 + 46X_7 + 55X_8$$

subject to the following constraints:

$$110X_2 + 1420X_3 + 2410X_4 \leq 2410$$

$$110X_6 + 2010X_7 + 2500X_8 \leq 2500$$

$$110X_2 + 1420X_3 + 2410X_4 \geq 110$$

$$110X_6 + 2010X_7 + 2500X_8 \geq 110$$

$$X_1 + X_2 + X_3 + X_4 = 1$$

$$X_5 + X_6 + X_7 + X_8 = 1$$

$$110X_2 + 1420X_3 + 2410X_4 + 110X_6 + 2010X_7 + 2500X_8 = Q$$

$$X_i \geq 0, \qquad i = 1,2,\ldots,8.$$

$(X_{i-3}, X_{i-2}, X_{i-1}, X_i)$ is a special ordered set.

The optimal discharge allocation and power output are found by solving the above linear programming model with specified values of Q. The feasible solution is determined by using branch-and-bound approach, as illustrated in Appendix I. The results are shown in Table 1.

TABLE 1

Optimal Discharge Allocation for a 2-Unit Hydro-Electric Generating Station

Total Discharge, Q (cu ft/sec)	Optimal Discharge Allocation (cu ft/sec)		Approximate Optimal Power Output (MW)
	Y_1	Y_2	
4500	2000	2500	95.8
4000	1500	2500	88.2
3500	1420	2080	79.3
3000	1420	1580	67.6
2500	1420	1080	55.5
2000	0	2000	45.8
1500	0	1500	33.7
1000	0	1000	21.5
500	0	500	9.4
200	0	200	2.2

3. OPTIMAL DISCHARGE ALLOCATION FOR IDENTICAL UNITS

This section concerns the division of discharge among identical generating units of a hydro-electric generation station to obtain maximum power output.

3.1 The Model

Since all the generating units have the same output-discharge curve, the notations used in the previous section can be simplified. Let $M_i = M$, $E_i = E$, $L_i = L$, $K_i = K$, and $P_i(Y_i) = P(Y_i)$ for $i = 1,2,...N$.

The nonlinear programming model can be written as follows:

Find $(Y_1, Y_2, Y_3, ..., Y_N)$ so that

$$\sum_{i=1}^{N} P(Y_i) \text{ is a maximum, subject to the constraints:}$$

$$L \leq Y_i \leq M \qquad i = 1,2,...,N$$

$$\sum_{i=1}^{N} Y_i = Q \qquad Q \leq NM$$

$$Y_i \geq 0 \qquad i = 1,2,...,N$$

This model can be solved using either the linear programming approach proposed in Section 2 or the method provided by Bernholtz/1/.

Bernholtz's approach calls for a set of rules for progressively loading up the generating units in a station. He proved mathematically that, by following these rules, optimal allocation of discharge will be achieved. His method can be used to build up discharge allocation tables, starting from Q=0, for generating stations. It cannot be used directly to obtain allocations for a specific value of Q without knowledge of the allocations for discharges less than Q.

In this section, a heuristic approach is proposed for the solution of the allocation problem. It is simple in concept and easy to program. The output-discharge curve of the generating units is required as before. If an equation for the output-discharge curve is not available, a piecewise-linear approximation of the following form will be needed:

$$\hat{P}(Y) = \begin{cases} 0 & \text{if } Y < L \\ P(L) + [P(E)-P(L)](Y-L)/(E-L) & \text{if } L \leq Y < E \\ P(E) + [P(M)-P(E)](Y-E)/(M-E) & \text{if } E \leq Y \leq M \end{cases}$$

where L, E, M, P(L), P(E) and P(M) are known.

The approach consists of three main steps:

Step 1: Determine the minimum (n_1) and maximum (n_2) number of generating units to be operated.

$$n_1 = [Q/M],$$

$$n_2 = \text{Min } \{[Q/L], N\}$$

where $[\cdot]$ denotes the largest integer less than the argument.

Step 2: For $j = n_1, n_1 + 1, n_1 + 2, \ldots, n_2$, compute the resulting power output from allocating the total charge Q among j generating units.

Step 3: Find the discharge allocation in Step (2) which gives the power output.

The approach is described by the flowchart shown in Figure 2. It will directly provide the maximum power output and discharge allocation. Note that the optimal allocation may involve less than total discharge, leaving a part of it unallocated. Unallocated discharge occurs because of the operating constraints of minimum and maximum discharges for each unit.

3.2 Example

Consider a hydro-electric generating station with the following characteristics:

$N = 3,$

$L = 233.8$ cu m/sec, $E = 273.0$ cu m/sec, $M = 314.6$ cu m/sec,

$P(L) = 45$ MW, $P(E) = 56$ MW, and $P(M) = 60$ MW.

For a specified total discharge through the station, the maximum power output is determined using the heuristic approach. The results are shown in Table 2 and Figure 3.

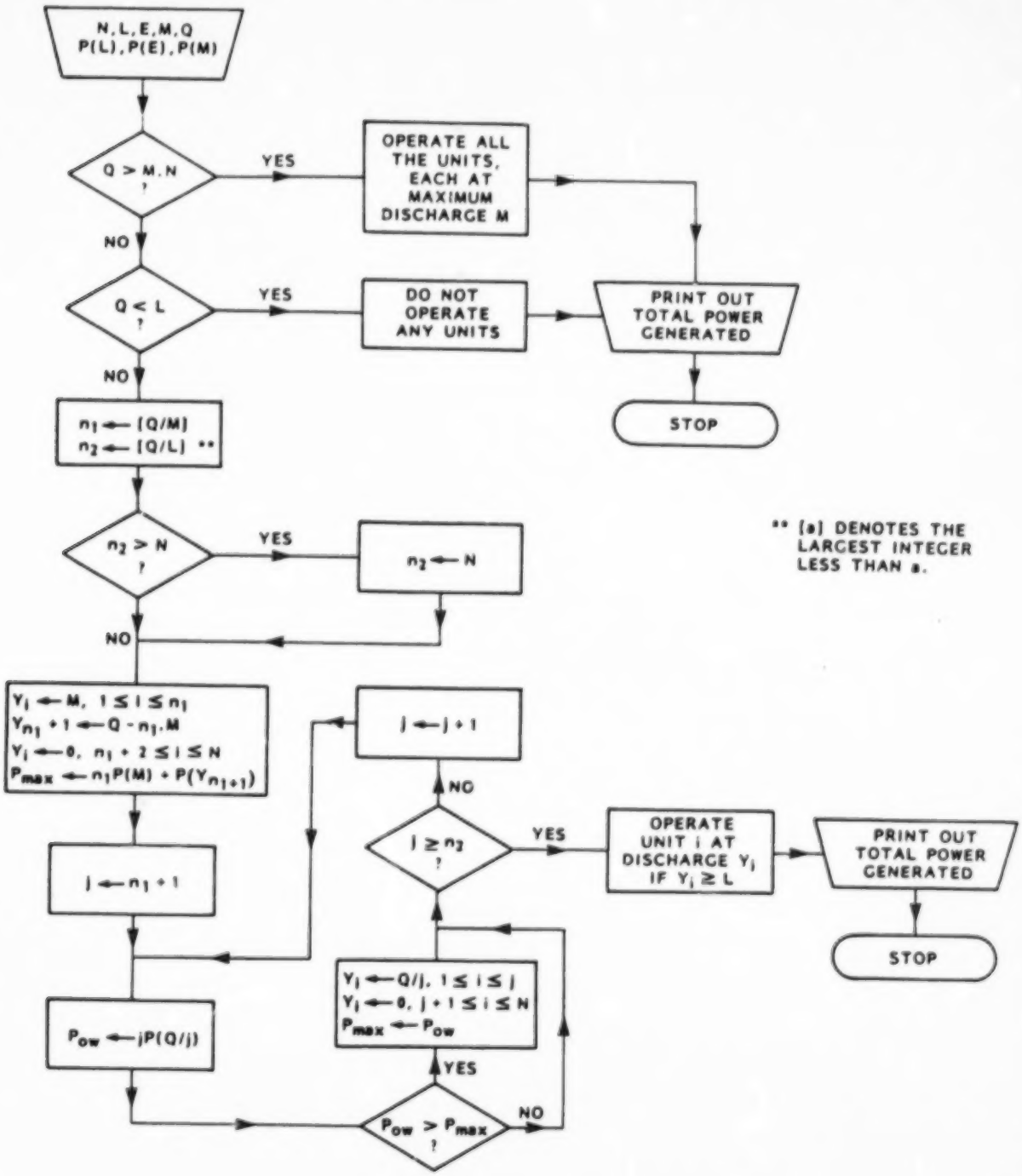

FIGURE 2

HEURISTIC APPROACH FOR ALLOCATING DISCHARGE AMONG IDENTICAL UNITS

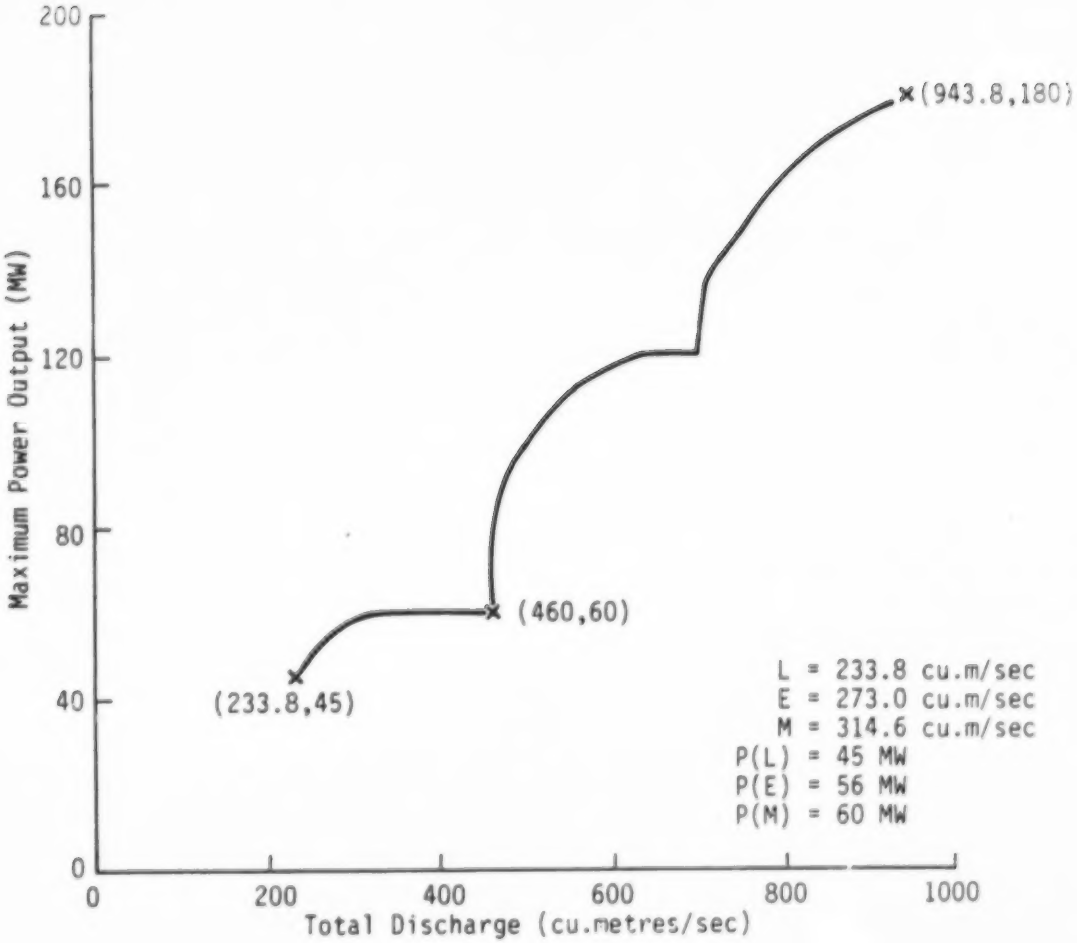

FIGURE 3

POWER OUTPUT OF A SYSTEM
OF THREE IDENTICAL GENERATING UNITS

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TABLE 2

Optimal Discharge Allocation for a Hydro-Electric Generating Station Consisting of 3 Identical Units

Total Discharge, Q (cu metres/sec)	Optimal Discharge Allocation (cu metres/sec)				Approximate Optimal Power Output (MW)
	Y_1	Y_2	Y_3	Unallocated	
200	0.0	0.0	0.0	200.0	0.0
250	250.0	0.0	0.0	0.0	49.5
300	300.0	0.0	0.0	0.0	58.6
350	314.6	0.0	0.0	35.4	60.0
400	314.6	0.0	0.0	85.4	60.0
450	314.6	0.0	0.0	135.4	60.0
500	250.0	250.0	0.0	0.0	99.1
550	275.0	275.0	0.0	0.0	112.4
600	300.0	300.0	0.0	0.0	117.2
650	314.6	314.6	0.0	20.8	120.0
700	314.6	314.6	0.0	70.8	120.0
750	250.0	250.0	250.0	0.0	148.6
800	266.7	266.7	266.7	0.0	162.7
850	283.3	283.3	283.3	0.0	171.0
900	300.0	300.0	300.0	0.0	175.8
950	314.6	314.6	314.6	6.2	180.0

4. DISCUSSION

In this preliminary study, methods for the division of discharge among individual hydraulic units in a hydro-electric generating station to achieve maximum power output are proposed.

A nonlinear programming model has been formulated for the discharge allocation problem. This is applicable to a hydro-electric generating station regardless of whether individual generating units are similar or not. The model can be solved using either nonlinear programming or linear programming software packages. No nonlinear programming software package is currently available in Ontario Hydro. However, it is commercially available. Linear programming software packages are available on Ontario Hydro's IBM and UNISYS (Sperry) mainframes. However, only the SAS-LP software package is reliable.

A heuristic approach has also been proposed for the discharge allocation problem if the generating units of the station are all identical.

In practice, the generating units in a hydraulic station may not be either all different or all alike but may be divided into groups of identical units. In addition to the methods mentioned earlier, an alternative approach for this case may consist of the following:

STEP 1: Determine the optimal mode of operation for each group of identical units by a heuristic approach (as in Section 3). That is, the maximum power output from a group of identical units is determined for specified discharges. An output-discharge curve for this "super unit" is thus be obtained.

STEP 2: Determine the division of the station discharge between groups (or super units) so as to produce maximum station output. The linear programming approach (in Section 2) may be suitable.

Since the discharge for each group is known, the actual division of discharge among identical units in the group can be easily obtained from Step 1.

5. RECOMMENDATIONS

1. Further work should be carried out to consider the suitability of using the various solution approaches proposed in this study for application to the hydraulic generating stations under given operating constraints. Refinement of the solution approaches should be carried out if necessary.

2. The use of linear programming software packages on either the IBM and UNISYS (Sperry) mainframes may not be very cost-effective. Considerations should be given to the evaluation and the purchase of quality software packages in linear and nonlinear programming which are suitable for personal computers/6/.

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APPENDIX I

BRANCH-AND-BOUND SOLUTION

In this appendix, the discharge allocation problem in Section 2.3 is solved for total discharge Q=3000. The branch-and-bound approach is used to determine the feasible optimal discharge allocation.

Problem

Maximize $Z=P_1(Y_1)+P_2(Y_2)$ subject to the constraints:

$$110 \leq Y_1 < 2410$$

$$110 \leq Y_2 \leq 2500$$

$$Y_1+Y_2=3000$$

$$Y_1,\ Y_2 \geq 0$$

Reformulated Problem Based on Piecewise-Linear Approximation

Maximize $Z=32X_3+47X_4+46X_7+55X_8$ subject to the constraints:

$$110X_2+1420X_3+2410X_4 \geq 110$$

$$110X_2+1420X_3+2410X_4 \leq 2410$$

$$110X_6+2010X_7+2500X_8 \geq 110$$

$$110X_6+2010X_7+2500X_8 \leq 2500$$

$$X_1+X_2+X_3+X_4=1$$

$$X_5+X_6+X_7+X_8=1$$

$$110X_2+1420X_3+2410X_4+110X_6+2010X_7+2500X_8=3000$$

$X_i \geq 0$, $i=1,2,\ldots,8$.

$(X_{i-3},X_{i-2},X_{i-1},X_i)$ is a special ordered set (SOS).

Solution

Step 1: Solve the LP problem (with the SOS requirement relaxed). The optimal solution is:

$X_1=0.3028$, $X_2=0$, $X_3=0.6972$, $X_4=0$,

$X_5=0$, $X_6=0$, $X_7=1$, $X_8=0$,

$Z_0=63.31$.

Note that X_1 and X_3 are non-zero, but they are not adjacent and so do not satisfy the SOS requirements. Now, any SOS solution must have either $X_1=0$ or $X_3=0$.

Step 2: Create LP problem 1 by setting $X_1=0$ in the original LP formulation.

The optimal solution is:

$X_1=0$, $X_2=0$, $X_3=1$, $X_4=0$,

$X_5=0.21393$, $X_6=0$, $X_7=0.78607$, $X_8=0$,

$Z_1=68.16$.

Note that X_5 and X_7 are non-zero, but they are not adjacent, and so do not satisfy the SOS requirements.

Step 3: Create LP problem 2 by setting $X_3=0$ in original LP formulation.

The optimal solution is:

$X_1=0.5892$, $X_2=0$, $X_3=0$, $X_4=0.4108$,

$X_5=0$, $X_6=0$, $X_7=1$, $X_8=0$,

$Z_2=65.31$.

Note that X_1 and X_4 are non-zero, but they are not adjacent, and so do not satisfy the SOS requirements.

Step 4: Create LP problem 3 by setting $X_5=0$ in LP problem 1.

The optimal solution is:

$X_1=0$, $X_2=0$, $X_3=1$, $X_4=0$,

$X_5=0$, $X_6=0.226316$, $X_7=0.773684$, $X_8=0$,

$Z_3=67.59$.

This is a feasible optimal solution since the SOS requirements are satisfied. Hence,

$Y_1=1420$, $Y_2=1580$.

Step 5: Determine whether there are any other feasible optimal solution by setting $X_1=0$ in LP problem 2, and so on.

Step 6: Compare all the optimal feasible solutions. The optimal solution is the one that gives the greatest power output.

For a good discussion of the Branch-and-Bound approach, see reference/3/.

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